**Properties of Addition**

|  |  |  |
| --- | --- | --- |
| **Property** | **Definition** | **Example** |
| Additive Identity  Element | The sum of a number and  zero is that number. |  |
| Commutative Property |  |  |
| Associative Property |  |  |
| Additive Inverses  (opposites) | The sum of a number and its opposite is 0. |  |
| Closure Property | The sum of any two numbers is a unique real number. |  |

**Properties of Multiplication**

|  |  |  |
| --- | --- | --- |
| **Property** | **Definition** | **Example** |
| Multiplicative Identity  Element | The product of a number and  one is that number. |  |
| Commutative Property |  |  |
| Associative Property |  |  |
| Multiplicative Inverses (Reciprocals) | When the product of two numbers is one. |  |
| Multiplication Property of Zero |  |  |
| Closure Property | The product of any two numbers is a unique real number |  |

Lesson 7A: Algebraic Expressions—The Commutative and Associative Properties

Classwork

**Four Properties of Arithmetic:**

The Commutative Property of Addition: If and are real numbers, then .

The Associative Property of Addition: If and are real numbers, then

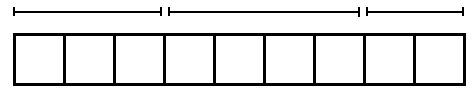
The Commutative Property of Multiplication: If and are real numbers, then .

The Associative Property of Multiplication: If and are real numbers, then .

Exercise 1

Viewing the diagram below from two different perspectives illustrates that equals

.

****

Is it true for all real numbers and that should equal ?

(Note: The direct application of the Associative Property of Addition only gives.)

Exercise 2

Draw a flow diagram and use it to prove that for all real numbers and .

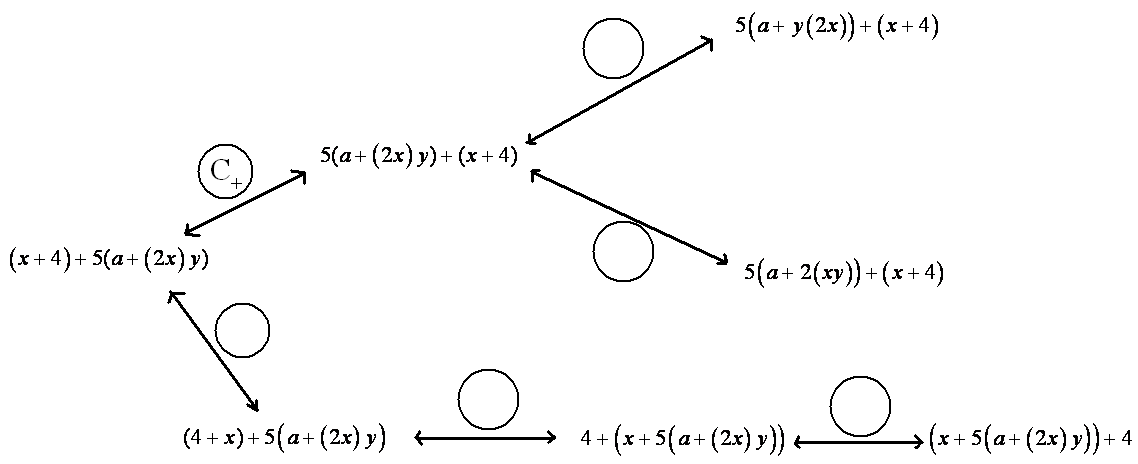
Exercise 3

Use these abbreviations for the properties of real numbers and complete the flow diagram.

for the commutative property of addition

for the commutative property of multiplication

for the associative property of addition

for the associative property of multiplication 

Exercise 4

Let and be real numbers. Fill in the missing term of the following diagram to show that is sure to equal .



**Important Definitions**

**NUMERICAL SYMBOL**: A *numerical symbol* is a symbol that represents a specific number.

For example, are numerical symbols used to represent specific points on the real number line.

**VARIABLE SYMBOL**: A *variable symbol* is a symbol that is a placeholder for a number.

It is possible that a question may restrict the type of number that a placeholder might permit, e.g., integers only or positive real number.

**ALGEBRAIC EXPRESSION**: An *algebraic expression* is either

1. a numerical symbol or a variable symbol, or
2. the result of placing previously generated algebraic expressions into the two blanks of one of the four operators or into the base blank of an exponentiation with exponent that is a rational number.

Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the Commutative, Associative, and Distributive Properties and the properties of rational exponents to components of the first expression.

**NUMERICAL EXPRESSION**: A *numerical expression* is an algebraic expression that contains only numerical symbols (no variable symbols), which evaluate to a single number.

The expression, , is not a numerical expression.

**EQUIVALENT NUMERICAL EXPRESSIONS**: Two numerical expressions are *equivalent* if they evaluate to the same number.

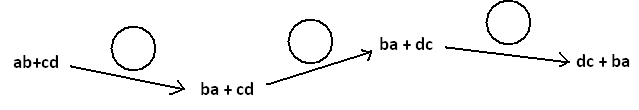
Note that and , for example, are equivalent numerical expressions (they are both ) but and are not equivalent expressions.

Lesson Summary

The Commutative and Associative Properties represent key beliefs about the arithmetic of real numbers. These properties can be applied to algebraic expressions using variables that represent real numbers.

Two algebraic expressions are ***equivalent*** if we can convert one expression into the other by repeatedly applying the Commutative, Associative, and Distributive Properties and the properties of rational exponents to components of the first expression.

Problem Set

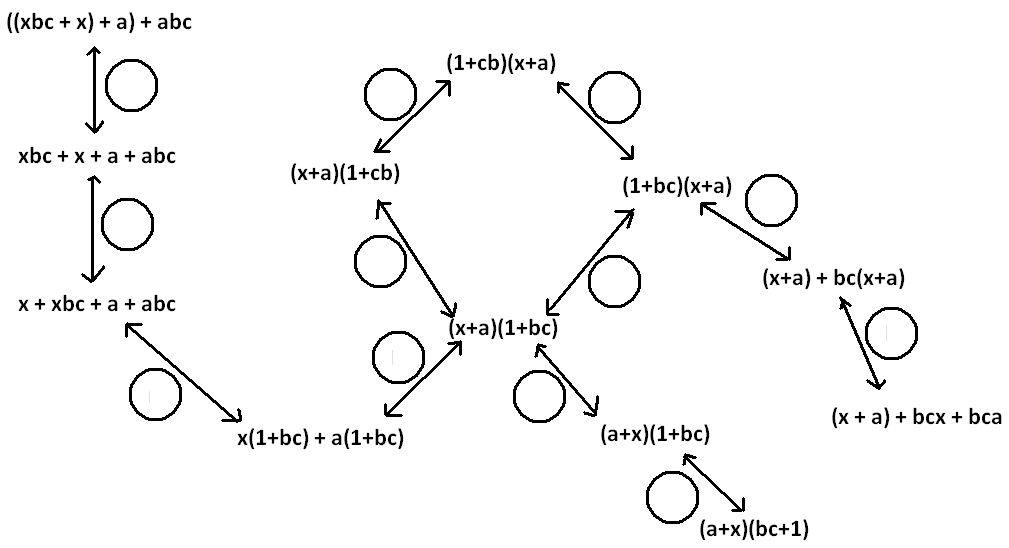
1. The following portion of a flow diagram shows that the expression is equivalent to the expression .

Fill in each circle with the appropriate symbol: Either (for the “Commutative Property of Addition”) or (for the “Commutative Property of Multiplication”).

1. Fill in the blanks of this proof showing that is equivalent . Write either “Commutative Property,” “Associative Property,” or “Distributive Property” in each blank.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. What is a quick way to see that the value of the sum is ?
2. Fill in each circle of the following flow diagram with one of the letters: C for Commutative Property (for either addition or multiplication), A for Associative Property (for either addition or multiplication), or D for Distributive Property.



1. The following is a proof of the algebraic equivalency of and . Fill in each of the blanks with either the statement “Commutative Property” or “Associative Property.”

Power to a Power\_\_\_\_\_\_\_\_\_\_\_

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Write a mathematical proof of the algebraic equivalency of and . (I provided a start for you there are three more steps to the conclusion)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. a. Suppose we are to play the -number game with the symbols a, b, c, and d to represent numbers, each used at most once, combined by the operation of addition ONLY. If we acknowledge that parentheses are not needed, show there are essentially only expressions one can write.

b. How many answers are there for the multiplication ONLY version of this game?

1. Write a mathematical proof to show that is equivalent to .

Lesson 7A: Operations on Algebraic Expressions

Recall the following rules of exponents:

|  |  |
| --- | --- |
| Product of  Powers | For any number *a*, and all integers *m* and *n*,  am ⋅ an = am+n |
| Power of  a Power | For any number *a*, and all integers *m* and *n*,  (am)n = amn |
| Power of  a Product | For all numbers *a* and *b*, and any integer *m*,  (ab)m = ambm |
| Power of  a Monomial | For all numbers *a* and *b*, and all integers *m*, *n*, and *p*,  (ambn)p = ampbnp |
| Quotient of  Powers | For all integers *m* and *n*, and any nonzero number *a*,  am ÷ an = am - n |
| Zero  Exponent | For any nonzero number *a*,  a0 = 1 |
| Negative  Exponents | For any nonzero number *a* and any integer *n*,  a-n = 1  an |

Study the examples below.

(83)5 = (83)(83)(83)(83)(83) product of powers (y7)3 = (y7)(y7)(y7)

= 83 + 3 + 3+ 3 + 3  = y7 + 7 + 7

= 815 = y21

85 = (8)(8)(8)(8)(8) y3 = **\_**yyy\_ = 1 or y-4

83  (8)(8)(8) y7 yyyyyyy y4

= 82

A **Monomial** is in **Simplest Form** when:

* There are no powers of powers
* Each base appears once
* All fractions are in simplest form
* No negative exponents

Simplify each expression

a2a3 = a2a4 = a2a5 = a4a7a =

(3c2)(2c3) = 54⋅57 = (xy4)(x2x3) = (8y6)(9y7) =

(-5a4)(6a) = (2x4)(4x3y2)(-3xy3) = (a4)2(b7)2(c)3 =

(63)5 = (z2y3)2 = (-2a2bc)3 =

(-6 • 5)2 = (4xy)3 = (3x4)2 • x5 = (-9)(-9z5)2 =

**Dividing Monomials**

y4 ÷ y1 z3 ÷ z2 x3 ÷ x3

105 ÷ 103 25a5 ÷ 5a2 x3 ÷ x4

Problem Set

**Write the expression as a single power of the base.**

1. 42 • 43 2. (-3)2(-3) 3. x • x2  4. a2a3a4

5. (42)3  6. [(-3)5]2 7. (a2)5 8. (x3)3

**Simplify the expressions.**

1. (-3 • 2)2 2. (2 • 4)3 3. (2w)6 4. (4y2z)3

**Write the expression as a single power of the base.**

1. a2a3 = 2. (c2c3)3 = 3. (x)(x3) = 4. 54⋅57 =

5. (y6)(y7) = 6. (-5)4(-5)2 = 7. x4x3x = 8. (a2)2 =

**Complete the statement.**

1. w7w? = w10 2. (7?)5 = 715 3. [(-2)3]2 = -2?

**Transform each given expression into an equivalent one with a positive exponent**

1. 4-3  2. 6(3-3) 3. 10-1 4. 27 ÷ 2-3

5. 1 6. 3-6 ÷ 3-2 7. (5)-2  8. (x4)-3

2-5

9. 24y4  10. -65z3  11. -21a5b4 12. 12y2z2

6y 13z2 -3a4b -4y2z

13. 21a2b 14. 3ab 15. 24x3y4 16. - 6xy

3ab 3ab -6xy 6xy

17. Simplify